PDE control Dallas-style: Oil drilling & production

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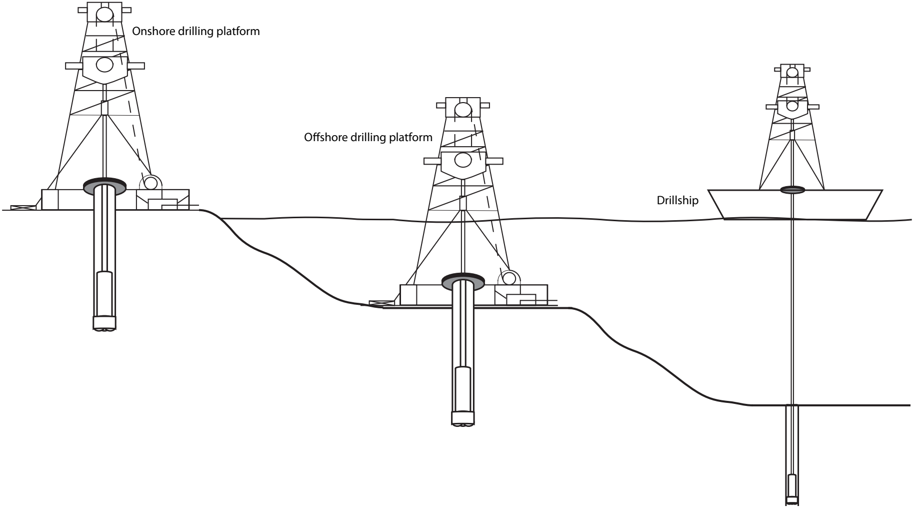
Spong Fest

Industrial setup



Length of a well: 100m–5km (300–15000 ft.)

Drill bit

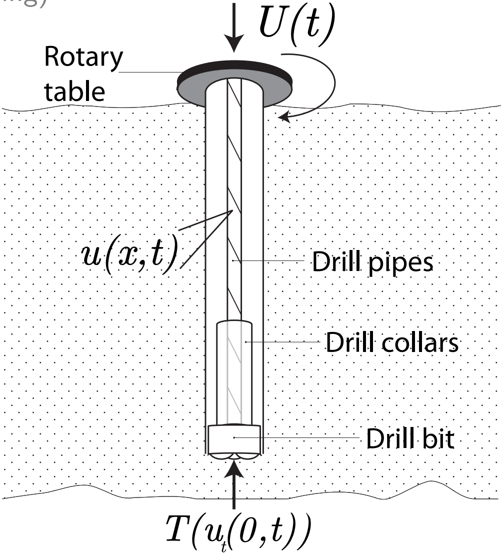
Common instabilities

Whirling oscillations (beam bending)

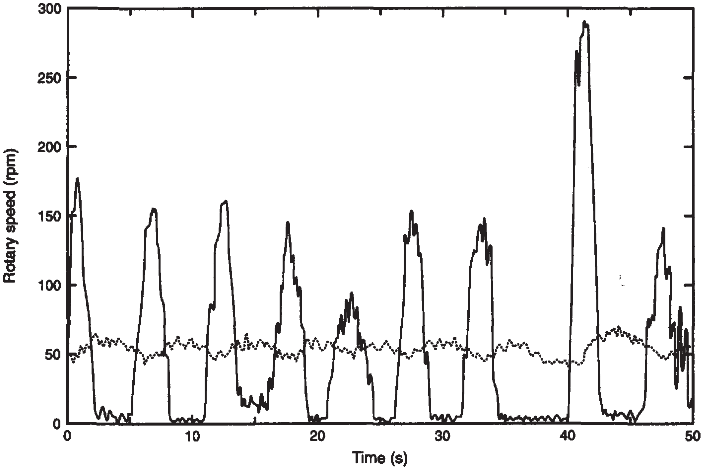
Vertical oscillations

- wave-induced mud pressure vibrations
- and "bit bounce" (w/ 3-cone bit)

Stick-slip oscillations (torsional)



Experimental data (stick-slip instability)



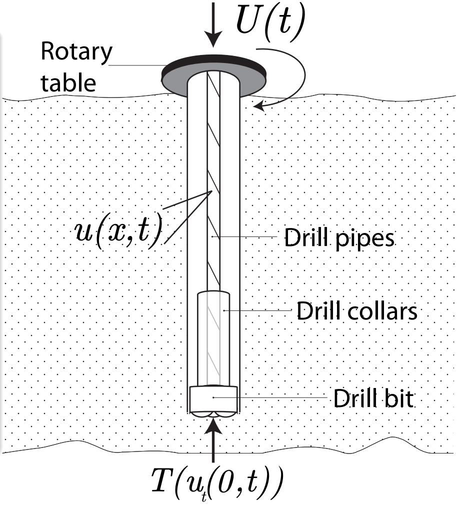
Model of angular displacement dynamics $u(x,t)$

Torsional wave equation + B.C.

$$u_{tt}(x,t) = u_{xx}(x,t)$$

$$u_x(1,t) = U(t) \qquad \text{(torque input)}$$

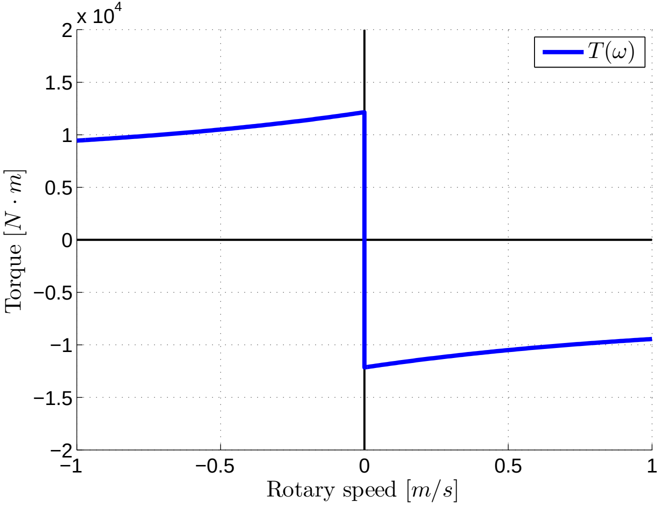
$$\underbrace{u_{tt}(0,t)}_{\text{bit accel}} = a\,\underbrace{u_x(0,t)}_{\text{drillstring force}} + a\,\underbrace{T\left(u_t(0,t)\right)}_{\text{friction force}}$$



Desired angular displacement trajectory for const. drill speed ω_r

$$\bar{u}(x,t) = \omega_r t - T(\omega_r)x + u_0 \qquad \bar{U} = -T(\omega_r)$$

Rock-on-bit friction (slope > 0 at higher speed)



Linearized model

PDE-ODE cascade (unstable)

$$
\begin{aligned}
u_{tt}(x) &= u_{xx}(x) \\
u_x(1) &= U(t) \\
u_{tt}(0) &= \underbrace{ab u_t(0)}_{\text{anti-damping}} + a u_x(0)
\end{aligned}
$$

where

$$b = \frac{\partial T}{\partial \omega}(\omega_r) > 0 \quad \text{(for large RPM)}$$

Reformulate:

drillstring torsional gradient (**twist**) + drill bit speed

- **twist**: $v(x,t) = u_x(x,t)$
- drill bit speed: $y(t) = u_t(0,t)$

New set of equations (still a PDE-ODE "cascade")

$$\begin{cases} u_{tt}(x) = u_{xx}(x) \\ u_x(1) = U(t) \quad \text{(Neumann)} \\ \\ u_{tt}(0) = abu_t(0) + au_x(0) \end{cases} \rightarrow \begin{cases} v_{tt}(x) = v_{xx}(x) \\ v(1) = U(t) \quad \text{(Dirichlet)} \\ v_x(0) = av(0) + \underbrace{aby(t)} \\ \dot{y}(t) = aby(t) + av(0) \end{cases}$$

Backstepping controller design

- stabilize bit
- free drillstring end from bit
- dampen the freed drillstring end

Target sys.

$$
\begin{aligned}
w_{tt}(x) &= w_{xx}(x) \\
w(1) &= 0 \\
w_x(0) &= c w_t(0) && \text{damper (torsional)} \\
\dot{y}(t) &= -\delta y(t) + a w(0) && \text{damper (kinetic friction)}
\end{aligned}
$$

Backstepping controller design (cont'd)

Transformation
$$w(x,t) = v(x,t) - \int_0^x k(x,\xi)v(\xi,t)d\xi - \int_0^x s(x,\xi)v_t(\xi,t)d\xi - \gamma(x)y(t)$$

Kernel ODE coupled w/ 2 Goursat PDEs on domain $\{0 \le \xi \le x \le 1\}$

$$\begin{aligned}
k_{xx}(x,\xi) &= k_{\xi\xi}(x,\xi) \\
\frac{d}{dx}k(x,x) &= 0 \\
k_\xi(x,0) &= ak(x,0) \\
&\quad + a^2 b\left[(s(x,0) - \gamma(x))\right] \\
k(0,0) &= a - c(ab+\delta)
\end{aligned}$$

$$\begin{aligned}
s_{xx}(x,\xi) &= s_{\xi\xi}(x,\xi) \\
\frac{d}{dx}s(x,x) &= 0 \\
s_\xi(x,0) &= as(x,0) - a\gamma(x) \\
s(0,0) &= -c
\end{aligned}$$

$$\begin{aligned}
\gamma''(x) &= abk(x,0) + a^2 b^2\left[s(x,0) - \gamma(x)\right] \\
\gamma(0) &= -(ab+\delta)/a \qquad \gamma'(0) = -(ab+\delta)bc
\end{aligned}$$

Explicit expressions for the kernels

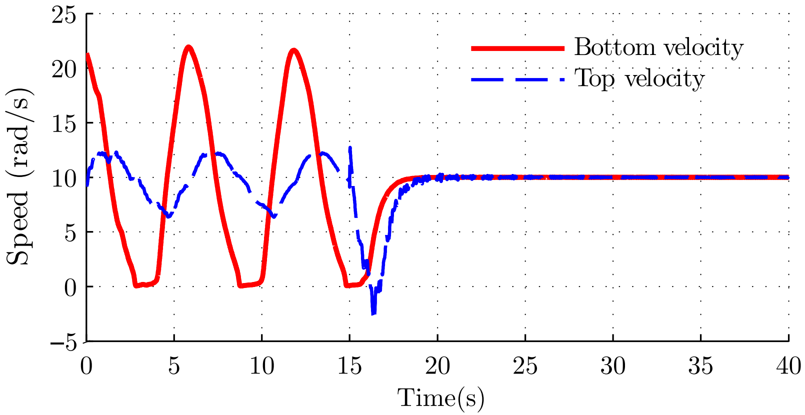
$$\begin{pmatrix} \kappa(x,y) \\ \sigma(x,y) \\ \gamma(x-y) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} e^{M(x-y)} \begin{pmatrix} a - c(ab+\delta) \\ -c \\ -(ab+\delta)/a \\ ab - cb(ab+\delta) \end{pmatrix}$$

$$M = \begin{pmatrix} -a & -a^2 b & a^2 b & 0 \\ 0 & -a & a & 0 \\ 0 & 0 & 0 & 1 \\ ab & a^2 b^2 & -a^2 b^2 & 0 \end{pmatrix}$$

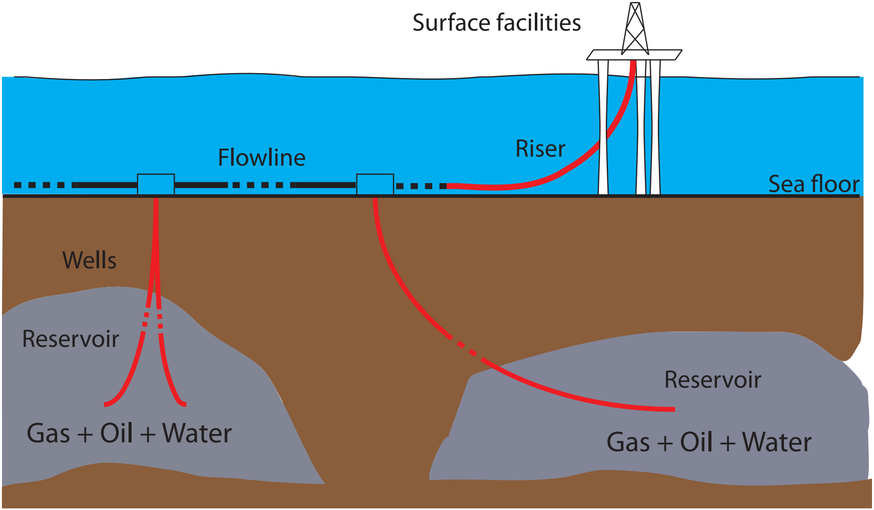
Feedback law (in original variables)—**Fancy PD controller**

$$U(t) = [a - c(ab+\delta)]\, u(1,t) - k(1,0)u(0,t) - \int_0^1 k_\xi(1,\xi)u(\xi,t)d\xi$$
$$+ c u_t(1,t) + [s(1,0) - \gamma(1)]\, u_t(0,t) + \int_0^1 s_\xi(1,\xi)u_t(\xi,t)d\xi$$

Simulations (control ON at 15 sec)

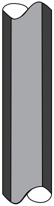


SLUGGING Flows in Offshore Oil PRODUCTION

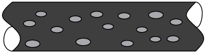


Two-phase (gas+liquid) flow regimes

Stratified



Annular



Bubbly

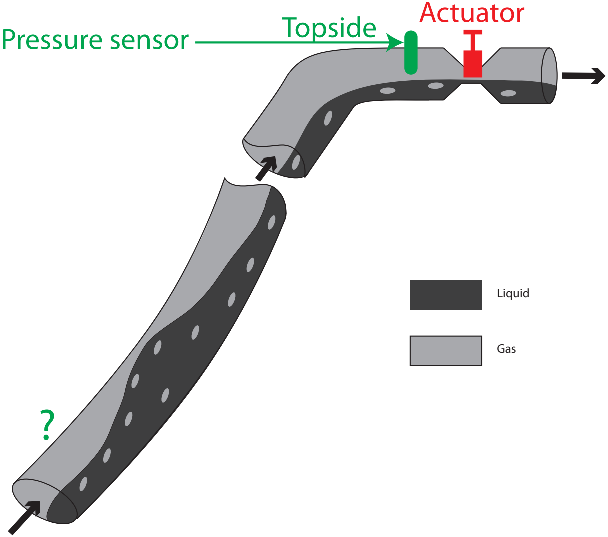
Steady flow

Slug

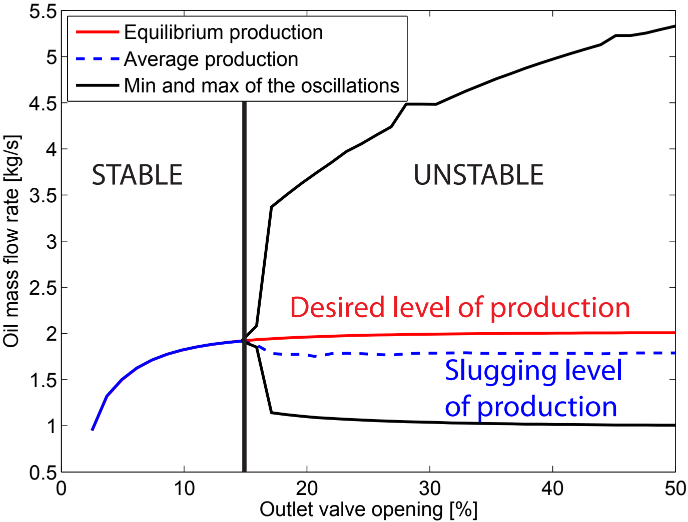
Periodic oscillations

Reduced production

Boundary control problem (with two sensing options)



Hopf bifurcation in production (unstable pneumatic spring)



Drift-flux modelling

Conservation of Mass & Momentum

$$\frac{\partial \alpha_G \rho_G}{\partial t} + \frac{\partial \alpha_G \rho_G v_G}{\partial z} = 0 \qquad \text{Mass of gas}$$

$$\frac{\partial \alpha_L \rho_L}{\partial t} + \frac{\partial \alpha_L \rho_L v_L}{\partial z} = 0 \qquad \text{Mass of liquid}$$

$$\frac{\partial \alpha_G \rho_G v_G + \alpha_L \rho_L v_L}{\partial t} + \frac{\partial P + \alpha_G \rho_G v_G^2 + \alpha_L \rho_L v_L^2}{\partial z} = -\rho_m g \sin\theta(z)$$

Combined momentum equation

Algebraic equations

- Closure relations: ideal gas law, slip relation,...
- Boundary conditions: Constant gas inflow, valve equation,...

Convert to Riemann variables

3 quasilinear transport eqns:
$w = (u_1, u_2, v) =$ (gas mass fraction, pressure, gas velocity)

$$\frac{\partial w}{\partial t} + A(w)\frac{\partial w}{\partial z} = S(w)$$

$\forall w$, $A(w)$ has 3 distinct eigenvalues

Physical interpretation

- u_1: pure transport [Riemann invariant] (ca. meters per second)
- u_2 and v: acoustic waves (both convection speeds ≈ 300 m/s)

Linearization around an equilibrium profile $\bar{w}(x)$

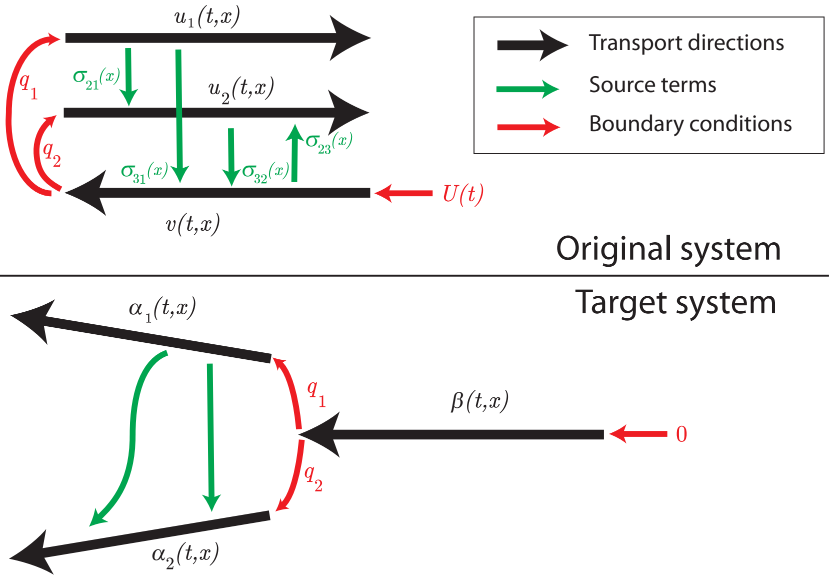
PDE

$$\underbrace{\begin{pmatrix} u_1 \\ u_2 \\ v \end{pmatrix}_t + \underbrace{\begin{pmatrix} \lambda_1(x) & 0 & 0 \\ 0 & \lambda_2(x) & 0 \\ 0 & 0 & -\mu(x) \end{pmatrix}}_{\Lambda(x)} \begin{pmatrix} u_1 \\ u_2 \\ v \end{pmatrix}_x}_{\text{exp. stable}} + \begin{pmatrix} 0 & 0 & 0 \\ \sigma_{2,1}(x) & 0 & \sigma_{2,3}(x) \\ \sigma_{3,1}(x) & \sigma_{3,2}(x) & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ v \end{pmatrix} = 0$$

Boundary conditions

$$\begin{pmatrix} u_1(0,t) \\ u_2(0,t) \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} v(0,t) \qquad\qquad v(L,t) = U(t)$$

System structure and stabilization strategy



Backstepping design

Target system: $$\underbrace{\boxed{\gamma_t + \Lambda(x)\gamma_x}}_{\text{exp. stable}} + \underbrace{\int_0^x C(x,\xi)\gamma(t,\xi)d\xi}_{\text{strict feedforward}} = 0$$

$$C(x,\xi) = \begin{pmatrix} 0 & 0 & 0 \\ \sigma_{2,1}(\xi)\delta(x-\xi) + c(x,\xi) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Backstepping transformation: $\gamma(t,x) = w(t,x) - \int_0^x K(x,\xi)w(t,\xi)d\xi$

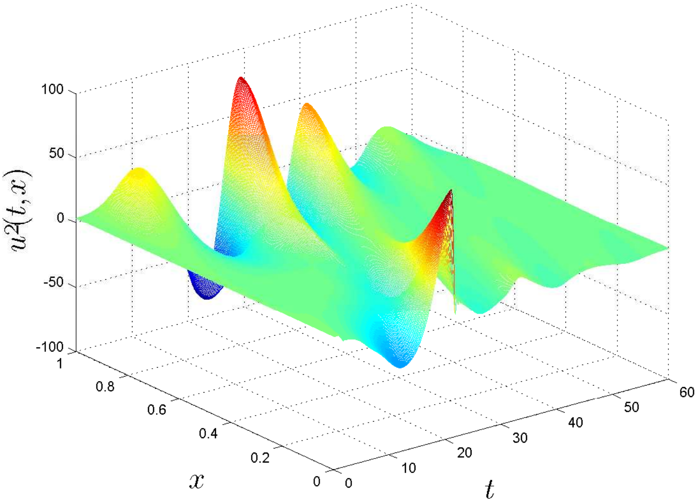
$$K(x,\xi) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & k^{2,2}(x,\xi) & k^{2,3}(x,\xi) \\ k^{3,1}(x,\xi) & k^{3,2}(x,\xi) & k^{3,3}(x,\xi) \end{pmatrix}$$

Control law (actuate gas flow via choke):

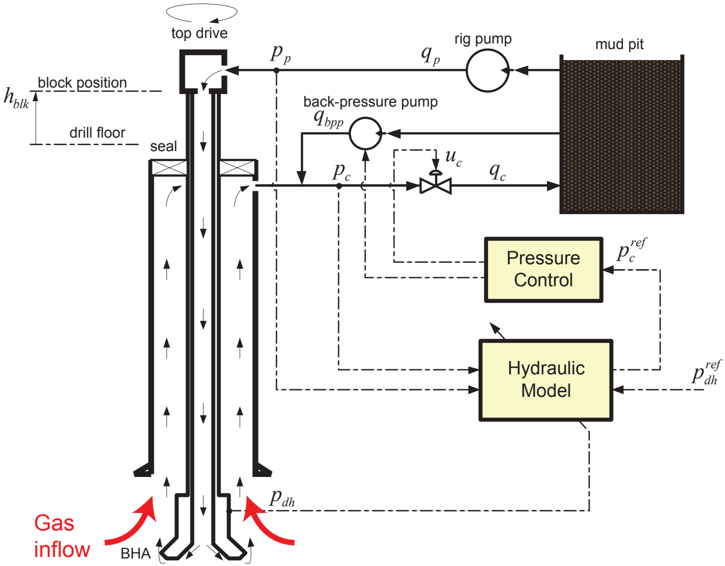
$$U(t) = \int_0^1 k^{31}(1,\xi)u_1(t,\xi) + k^{32}(1,\xi)u_2(t,\xi) + k^{33}(1,\xi)v(t,\xi)d\xi$$

Observer-based control (ON at $t = 20$ s): shown: gas pressure $u_2(t, x)$

actuation: flow rate at top $v(t, L)$; measurement: flow rate at bottom $v(t, 0)$



Mud-assisted drilling (helps take cuttings away)



Multiphase flow during drilling

- **Mud circulates** from mud pit, down through drill string, up through casing, and back into mud pit, controlled by choke valve
- Goal: regulate mud pressure to

$$\text{“collapse”} < \underbrace{\textbf{mud pressure} < \text{“pore”}} < \text{“fracture”}$$

  to help **gas** get ingested into casing (increases penetration rate)
- Model (physical): separate momentum and mass conservation laws for gas and mud/rock
- Model (in Riemann variables): 2 positive characteristic speeds for mass transport + 2 characteristic speeds for pressure waves (one positive, one negative)
- $\mathbf{(3{+}1)}\times\mathbf{(3{+}1)}$ hyperbolic system

Extension to $(n+1) \times (n+1)$ systems